3.28) a) $q_0 = 1$, $q_1 = x$, $q_2 = x^2$

$B = \{\underbrace{q_0}_{v_1}, \underbrace{q_1}_{v_2}, \underbrace{q_2}_{v_3}\}$. Busco uma base ortonormal $B'$ t.q. $B' = \{\underbrace{p_0}_{u_1}, \underbrace{p_1}_{u_2}, \underbrace{p_2}_{u_3}\}$

1- $k = 1$

2- $u_1 = \frac{v_1}{\|v_1\|}$ I $\rightarrow$ $\boxed{u_1 = \frac{1}{\sqrt{2}}}$

3- $U = \{u_1\}$

4- ... $k = 1$

5- $w_2 = v_2 - \langle v_2, u_1 \rangle u_1$ II $\rightarrow$ $w_2 = x$

6- $u_2 = \frac{w_2}{\|w_2\|}$ III $\rightarrow$ $\boxed{u_2 = \frac{x}{\sqrt{\frac{2}{3}}}}$

7- $U = \{u_1, u_2\}$

4- ... $k = 2$

5- $w_3 = v_3 - \langle v_3, u_1 \rangle u_1 - \langle v_3, u_2 \rangle u_2$ IV $\rightarrow$ $w_3 = x^2 - \frac{1}{3}$

6- $u_3 = \frac{w_3}{\|w_3\|}$ V $\rightarrow$ $\boxed{u_3 = \frac{x^2 - 1/3}{\sqrt{\frac{8}{45}}}}$

$\rightarrow B' = \{u_1, u_2, u_3\}$

BASE ORTONORMAL DE $\mathbb{R}_2[x]$.

UTILIZO BASE ORTOGONAL HALLADA EN a)

b) $P_{\mathbb{R}_2[x]}(\operatorname{sen} x) = \dfrac{\langle \operatorname{sen} x, 1\rangle}{\|1\|^2} 1 + \dfrac{\langle \operatorname{sen} x, x\rangle}{\|x\|^2} x + \dfrac{\langle \operatorname{sen} x, x^2 - 1/3\rangle}{\|x^2\|^2} (x^2 - \frac{1}{3})$ VI

$\rightarrow \boxed{P_{\mathbb{R}_2[x]}(\operatorname{sen} x) = 3x(\operatorname{sen}(1) - \cos(1))}$

IDEM PROCED. P/ $P_{\mathbb{R}_2[x]}(\cos x)$

c) $d(\operatorname{sen} x, \mathbb{R}_2[x]) = \|\operatorname{sen} x - 3x(\operatorname{sen}(1) - \cos(1))\| =$

$= \|\operatorname{sen} x + 2{,}947\, x\|$ VII $\cong \boxed{9{,}89}$

IDEM PROCED P/ $d(\cos x, \mathbb{R}_2[x])$

# CALCULOS AUX

I $\dfrac{1}{\underbrace{\|1\|}_{}} \rightarrow \|1\|^2 = \langle 1,1\rangle \overset{\text{com PI dado}}{=} \int_{-1}^{1} 1 \cdot 1 \cdot dx = 2 \rightarrow \|1\| = \sqrt{2}$

$\rightarrow = \dfrac{1}{\sqrt{2}}$

II $x - \left\langle x, \frac{1}{\sqrt{2}}\right\rangle \cdot \frac{1}{\sqrt{2}} = x - \int_{-1}^{1} \frac{x}{\sqrt{2}}\, dx \cdot \frac{1}{\sqrt{2}} = x - \frac{1}{\sqrt{2}} \cdot \frac{x^2}{2}\Big]_{-1}^{1} \cdot \frac{1}{\sqrt{2}} =$

$= x - 0 = x$

III $\dfrac{x}{\sqrt{\langle x, x\rangle}} = \dfrac{x}{\sqrt{\int_{-1}^{1} x^2 dx}} = \dfrac{x}{\sqrt{\frac{2}{3}}}$

IV $x^2 - \underbrace{\left\langle x^2, \frac{1}{\sqrt{2}}\right\rangle}_{*1} \cdot \frac{1}{\sqrt{2}} - \underbrace{\left\langle x^2, \frac{x}{\sqrt{\frac{2}{3}}}\right\rangle}_{*2} \frac{x}{\sqrt{\frac{2}{3}}}$ $\Delta$

$*_1 \rightarrow \int_{-1}^{1} \frac{x^2}{\sqrt{2}}\, dx = \frac{1}{3\sqrt{2}} \cdot x^3\Big]_{-1}^{1} = \frac{2}{3\sqrt{2}}$

$*_2 \rightarrow \int_{-1}^{1} \frac{x^3}{\sqrt{\frac{2}{3}}} = \frac{1}{4\sqrt{\frac{2}{3}}} \cdot x^4\Big]_{-1}^{1} = 0$

$\Delta \rightarrow x^2 - \frac{1}{3} - 0 = x^2 - \frac{1}{3}$

V $\dfrac{x^2 - \frac{1}{3}}{\underbrace{\left\langle x^2 - \frac{1}{3}, x^2 - \frac{1}{3}\right\rangle}_{*3}}$ $\Delta_2$

$*3 \rightarrow \|w_3\|^2 = \int_{-1}^{1} \left(x^2 - \frac{1}{3}\right)^2 dx = \int_{-1}^{1} \left(x^4 - \frac{2}{3}x^2 + \frac{1}{9}\right) dx =$

$= \left(\frac{x^5}{5} - \frac{2}{9}x^3 + \frac{x}{9}\right)\Big]_{-1}^{1} = \left(\frac{4}{45}\right) - \left(\frac{-1}{5} + \frac{1}{9}\right) = \left(\frac{4}{45}\right) - \left(\frac{-4}{45}\right) = \frac{8}{45}$

$\rightarrow \|w_3\| = \sqrt{\frac{8}{45}}$

$\Delta_2 \rightarrow \dfrac{x^2 - \frac{1}{3}}{\sqrt{\frac{8}{45}}}$

$V_1 \rightarrow \dfrac{\int_{-1}^{1} \operatorname{sen} x\, dx \cdot 1}{2} + \dfrac{\int_{-1}^{1} x \operatorname{sen} x\, dx \cdot x}{\frac{2}{3}} + \dfrac{\int_{-1}^{1} \left(x^2 - \frac{1}{3}\right) \operatorname{sen} x\, dx \cdot \left(x^2 - \frac{1}{3}\right)}{\frac{2}{5}} =$

$= \cancel{\dfrac{-\cos x \Big]_{-1}^{1}}{2}}^{\nearrow 0} + \dfrac{\left[2(\operatorname{sen}(1) - \cos(1))\right]}{\frac{2}{3}} x + 0 =$

$= 3\left(x \operatorname{sen}(1) - x \cos(1)\right)$

$V_{11} \rightarrow \|\operatorname{sen} x + 2{,}947x\|^2 = \int_{-1}^{1} (\operatorname{sen} x + 2{,}947x) \cdot (\operatorname{sen} x + 2{,}947x)\, dx =$

$\cong 9{,}89$